**ABSTRACT**

This study examined students’ ability to interpret and derive meaning from graphs by conducting a simple survey about a graph in a real-world scenario. The intent is to see if our results support or conflict with the current body of knowledge, which suggests that though students are often able to define and even draw graphs, they frequently perform poorly on graph interpretation items. The sample for this study consisted of eighth to twelfth grade students in classes ranging from Algebra I to Calculus. Descriptive analyses, using SPSS and Excel, looked at the patterns of students’ errors and the nature of their construction. These are discussed along with their implications for instruction.

*Keywords*: graphs, interpretation, instruction

**Student Misconceptions about Graphs: A Simple Study**

**General Statement of the Problem**

The National Council of Teachers of Mathematics (NCTM, 1989) has called for an increased emphasis in the middle grades on (a) reasoning from graphs and (b) the description and representation of relationships with tables, graphs, and rules. NCTM has also called for an increased emphasis in high school on (a) the connections between problems, functions, and graphs and (b) the use of computer-based graphing utilities. These increased emphases on understanding and reasoning from graphs may be seen as a natural evolution based on available technology; one that fits an educational philosophy that emphasizes understanding over rote memorization processes, and accompanied by a decreased emphasis on the mechanics of plotting graphs with paper and pencil. Despite the availability of tools such as computers and graphing calculators, understanding and using graphs remains a source of difficulty for many students.

In this paperwe look at some of the research into students’ difficulties understanding and interpreting graphs, evaluate the results of a simple survey in a real-world situation to support or refute current views, and discuss the implications for teaching.

**Review of Related Literature**

In mathematics curricula, graphs appear primarily in two ways. First, they may be used in the context of measurement and statistics for the presentation and analyses of data. Many types of graphs can be used to represent data sets (such as bar graphs, line graphs, pie charts, etc.). Second, graphs may be used in the context of algebra for the visual representation of functions; these are usually line graphs in the middle grades.

When used for statistics, graphs provide students many ways to organize, analyze, and present data. Nowadays, software tools simplify some of the mechanical aspects of graph construction and manipulation, and facilitate understanding by allowing students to spend more time interpreting graphs in just the ways called for by NCTM (1989). Additionally, students will have more time to explore different types of graphs, helping them to develop a stronger understanding through connecting data with various displays of that data. In algebraic situations, graphs and equations of functions play mirror roles—equations are the symbolic representation of a relationship, while graphs are the visual representation of that relationship. Traditionally, student tasks in this area involve creating graphs from functions, picking out features of graphs, and solving equations based on those graphs. Research suggests that many students have difficulty connecting graphs to functions.

Graphs and functions may also appear in the context of real-world situations, such as vehicular motion or the volume of water in a bathtub. In these instances, students may need to relate to the graph; not only to the symbolic representation of a function, but also to events in a real-world context. In both cases, it is important to point out that different types of graphical displays are appropriate in different situations. Students not only need to understand how to construct and interpret these graphs, but also how to make meaning from the graphs; understanding what each graph makes visible and which graph is most appropriate for a given situation.

**Students' Understandings and Misconceptions of Graphs**

**Graph Sense**

Factors associated with graph comprehension include prior knowledge of mathematics, the topic of the graph (i.e., what the data represent), and graphical form, such as the relationship between column height and quantity (Curcio, 1987). Friel, Bright, and Curcio (1997) used the term “graph sense” to describe students’ capacity to make sense of graphs and the data they represent. They identified three levels of graph sense:

1. Reading the data:

Students can answer literal questions about the specific data pictured in a graph. Reading the data requires that students understand what a graph is, what the axes represent, where the data are located, and what the data represent.

1. Reading between the data:

Students can identify and explain relationships in the data portrayed in the graph. This requires that students understand not only the basic structure of the graph (described above), but also make sense of the variation in values or types of data portrayed.

1. Reading beyond the data:

Students can extrapolate from the data and relationships portrayed in the graph. To do this students are required not only to understand the structure of the graph and the relationships contained therein, but also the context in which the data are presented. In other words, students must be able to use the information presented in the graph to answer questions that extend beyond the data contained in the graph.

These three levels of graph sense have been most commonly explored in employing a construction involving an elementary understanding of graphs based on simple extraction of the data from the graph, an intermediate level of understanding based on identifying relationships in the data, and an overall level of understanding based on extrapolating beyond the data contained in the graph to make broader judgments (Friel, Curcio, & Bright, 2001). The inference here is that students’ level of graph sense may be identified by asking specific types of questions in a particular sequence.

Entry level questions should focus on the literal content of the graph; the specific value of a data point or the meaning of the axes. Next, intermediate questions should address relationships between data points; changes over time or comparisons between elements (such as distance-time relationships on a distance-time graph). Then, overall questions should require predictions or inference based on the data presented.

While it may be possible to assess students’ level of “graphicacy,” Berg and Smith (1994) caution that the methodology and instruments used by researchers (and teachers) directly influence the outcomes observed, and found that when presented with graphical interpretation questions in multiple-choice format, students answered significantly more questions incorrectly than if they were asked to construct a graph themselves and then explain their reasons for doing so.

**Student Misconceptions of Graphs**

Researchers have noted that students have trouble making connections between graphs and other representations, whether those other representations are, for example, data sets, algebraic functions, real-world events, or other types of graphs. Friel and Bright (1996) note that in interpreting line plots, students sometimes confuse the number of *x*’s (or referents) associated with a particular value on the *x*-axis with the value itself.

In another study, Roth and Lee (2004)focused on the process of interpreting graphs that were unfamiliar and, “. . . concerned with understanding the processes by means of which graph interpretations were generated in real time, including how they start and stop” (p 268). This was accomplished by asking ‘*How do people asked to interpret unfamiliar graphs generate the requested interpretation, in real time, from moment to moment*?', *'At what point do they stop*?', and, *'What is the relation between the processes and products of interpretation and the research context in which these are produced*?’(p. 266). Their results seem to reflect a conclusion that, at the high school level, administering graphs with pertinent information to a student’s interests may result in a better interpretation of the unfamiliar graph as opposed to a graph with data that is completely irrelevant to a high school level mentality and experiences.

Along similar lines, several researchers have found that students have a tendency to think of a graph as a “picture” of the data rather than a representation of it. For example, in a graph showing a movement, students often confused a sloped line showing distance over time with the actual direction of movement, mistaking acceleration for movement in a northwesterly direction (Berg & Smith, 1994). This suggests that students fail to make the conceptual connection between the image of the graph and the data it represents.

**Line Graphs and Functions**

Several studies have identified line graphs as the most difficult for students to construct and interpret (Thomas, 1933; McDonaldRoss, 1977). Padilla, McKenzie, and Shaw (1986) identified sub skills associated with constructing and interpreting line graphs. To construct a line graph, students need to be able to draw and scale axes, assign “manipulated and responding variables” to each axis, plot points, and use a line of best fit. To interpret a line graph, they must be able to determine the *x* and *y* coordinates of a point, interpolate and extrapolate, state relationships between the variables, and correlate the results of multiple graphs. In assessing middle- and high-school students’ performance on each of these sub skills, the authors found that middle-school students were most successful in plotting points and determining the *x* and *y* coordinates of a point however, they struggled with scaling the axes and using a best fit line. The authors suggested that these difficulties may represent a lack of the mathematical knowledge required to develop these skills.

Hattikudur, Prather, Asquith, Alibali, Knuth, E. et al. (2012) analyzed the difficulties of students constructing graphs due to misconceptions about graphing by posing questions, such as: “(1) How successful are students at graphing slope and *y*-intercept, and how did this performance change with grade level, and (2) Do qualitative or quantitative features of a graph influence students’ ability to graph slope and *y*-intercept?” (p. 232). Their study revealed that “there was a significant interaction between grade level and concept” (p. 233). Overall, slope was correctly represented more than the *y*-intercept even in 6th grade where students had not been formally introduced to the concept but still had an “intuition about slope” (p. 234). Additionally, the data showed that there was an increase in the number of students that grasped the concept as the grade level increased. With respect to the second question posed, “the type of graph did not affect performance” (p. 234) when analyzing the slope component of the question. However, for graphing the “*y*-intercept […] sixth grade students had more difficulty when the graph was presented with qualitative features than with quantitative features” (p. 234). The study revealed that “middle school students have more difficulty graphing *y*-intercept than slope” (p. 239) regardless of the type of graph that was being presented.

Dunham and Osborne (1991) identified a number of common errors in student comprehension of graphs associated with functions, some of which may be generalized to graphs as a whole. First, they discovered that in looking at “points” on the graph of a function, students often fail to recognize them as ordered pairs— “projections” from the *x* and *y* axes. In other words, the value of a point within a function is not associated with its relative position along the axes in question. Second, students fail to understand that the “line” in a graph representing a function is actually made up of an infinite number of ordered pairs produced by that function. The authors speculate that because students learn to graph linear functions using the “slope-intercept” method (calculating the slope and the points at which the line intercepts that axes), students tend to view the line as connecting two or more fixed points. Finally, the authors noted that in reading graphs, students struggled with the concept of scale. Specifically, students could not explain how a change in the scale of one of the axes would affect the shape of the graph. Again, this suggests that students do not associate the shape of a graph (in this case, a function) with the data the graph was intended to represent as indicated by the labels and axes. At the middle school level or below, this confusion may reflect students’ lack of understanding of ratio and proportion—concepts central to understanding scale.

Knuth (2000) studied student understanding of the connection between functions and graphs. Given a question about a graph and related function, students tended to do complex calculations with the function in order to answer the question, rather than read the answer off the graph. He suggests that students have a limited understanding of the relationship between graphs and functions. He notes that while students often create graphs from functions, they rarely get practice creating functions from graphs.

Judit Moschkovich (1999) studied the nature of students’ use of the *x*-intercept of the lines of the form *y = mx + b* by summarizing the results of written assessments and presenting two case studies of students discovering and discussing linear equations and their graphs. Her research showed that her students did not have the clear logic on the relationship of the lines and the *x*-intercepts, and there were several uses of the *x*-intercept documented as the value of the *y*-intercept in relation with the slope to describe the rate of change of the line. This misconception was not just a simple mismatch with some principle of competent knowledge.

Mevarech and Kramarsky (1997) conducted a study to determine students’ conceptions and alternative conceptions relating to the construction of graphs that depict everyday situations, and to investigate the resistance of these alternative conceptions to formal instruction about graphing, based on the premise that (a) the correctness of the construction of a graph is affected by the students’ conceptions and alternative conceptions, and (b) prior knowledge is a key part in students’ learning. The study focused on the question, “*Would formal instruction about graphing change students’ alternative conceptions*?” They concluded that the alternative conceptions identified in the study stem from a) deficiencies in students’ information processing, negative transfer, poor understanding of the notion of co-variation, confusion between the process and the product, and difficulties attributed to misunderstanding or misuse of the language.

Vellom, R., & Pape, S. (2000) examined how the students thought about data sets and representing data with more than two variables, and how the representations of a particular student affected the thinking of other students in their class. Most of the students failed to make relational statements from their representations, and their graphs did not show a clear understanding of the difference between an independent and a dependent variable, and those graphs that attempted to depict more than two variables were inefficient and superficial.

Parmar and Signer (2005) looked at the performance of students with and without Learning Disabilities (LD) when creating graphs, their explanations while engaged in the tasks, and their ability to explain their understanding of a graph, and suggested that students’ difficulties during mathematics activities are influenced by the context of the task, the cognitive complexity of the activity, and the failure to attend to or correctly interpret various sources of information, and that although students with LD may understand concepts such as graph interpretation at a rudimentary level (e.g., using one variable and identifying information), they need more focused intervention in dealing with complex tasks, particularly those where the construction of a response is required.

Another issue that students often struggle with is the notion of linearity. For example, a student plotting a linear function may plot one point incorrectly and connect all of the plotted points with straight line segments that go from point to point. They do not realize, in looking at the graph as a whole, that it displays a nonlinear function. As a second example, students given a graph of a linear function in which the axes are unlabeled and for which they are asked to complete a table of values that could be represented by the function will often pick points that are not linear (e.g., (1,1), (2,2) (3,5), (6,6)).

**Research Question(s), Hypothesis, or Foreshadowed Problems**

This study was undertaken to find out whether or not the contention that students have problems and misconceptions about graphs is true. It is our belief that the information will indicate that, for the students surveyed, discrepancies and misconceptions to exist with regards to their understanding and interpretation of graphs. The survey contained both quantitative and qualitative aspects, and simple descriptive statistics were used to analyze the information gathered.

**Significance of the Proposed Study**

This study is important because, since 1989, the National Council of Teachers of Mathematics (NCTM) has advocated increased use of learning activities that stress mathematical communication through talking and writing. Graph construction and interpretation is a mathematical activity that lends itself to the attainment of the NCTM Process Standard.

**DESIGN/METHODOLOGY**

**Subjects and/or Case**

In this quantitative study respondents were taken from one comprehensive middle school and two comprehensive high schools in Southern, Ca. There were a total of 159 participants comprised of eighth, ninth, tenth, eleventh, and twelfth-grade students (82 females and 77 males). The students studied in one of the five mathematics courses offered at their school, Algebra I, Geometry, Algebra II, Pre-Calculus, or Calculus. Of the 159 participants, 60 were eighth-grade students from the middle school (32 Algebra I students and 28 Geometry students) and 99 were either ninth, tenth, eleventh, or twelfth-grade students from the high school (36 Algebra II students, 27 Pre-Calculus students, and 36 Calculus students). In addition, students in all three schools came from mixed socioeconomic families. For the purpose of this study it was pertinent to survey students from all five core mathematics courses offered in the public education school system. The rationale in doing so was that data would be collected to prove or disprove growth in the understanding of graphs as a student’s grade and level of mathematics increased.

**Instrumentation/Data Collection**

The instrument used for this study was a survey. The survey was administered by the students’ respective math teacher in the students’ respective math class. Subjects were asked to answer nine questions relating to a given graph. Students each received one survey copy, 8 x 5.75 in. On the top of the page students were asked to only provide their math class, grade level, and gender. No formal instruction was given prior to or after the students took the survey. Questions 1 to 4 assessed students’ knowledge of the labeling of a graph, questions 5 to 6 assessed students’ ability to understand Cartesian coordinates and intersection points, and questions 7 to 9 assessed students’ understanding of interpreting data from a graph. Of the nine questions only one question was a free response, question 7.

The graph itself was taken from the Common Core Smarter Balanced Assessment Consortium (See Appendix D); however, the nine questions are original and used for this study only. See appendix B for the survey that was administered for this study.

The advantages to using this survey are that the student responses will indicate the areas of difficulty a student has in understanding a situation presented through a graph. The situation, a “race”, represented in the graph is also very relatable to the students. However, if students’ have no prior knowledge of a “hurdle” this may impact the response to question 7. The answer to question 1 and 3 were purposely designed to be the same, as goes for question 2 and 4. These questions were worded differently, although they employed the same response. The purpose in doing so was to determine student discrepancies’ between the mathematical language used in labeling graphs, i.e. *x*-axis/independent variable and *y*-axis/dependent variable.

**Data Treatment Procedures**

The data of the present study was analyzed by four secondary education mathematics teachers. The analysis procedure involved the following steps: (a) classifying the survey questions into appropriate categories based on relatedness to graphic interpretation; (b) determining inter-rater agreement; and (c) examining each survey and deciding whether or not the responses were correct. Responses were considered correct/incorrect if they fell within the predetermined survey response key (See Appendix C). Each of the nine questions was scored as either 1 (correct) or 0 (incorrect). It should be noted that since question 7employed a free response, different interpretations of what happened to runner C were assigned a point of 1, if they met the understanding that runner C was at a halt approximately 13 seconds into the race. Data of each student surveyed were organized into two Excel spreadsheets according to their respective math course. The first spreadsheet categorized gender, grade level, response score (1 or 0) for each of the nine questions, and correct response distributed by gender/grade/math class. The second spreadsheet organized raw data for questions 5, 6, 8, and 9. This spreadsheet also displays the mean, median, and mode computed for the response data of questions 5, 6, and 9.

The data organized in the two spreadsheets was taken and converted into six tables and five pie charts. Table 1 displays the mean, median, mode, range, maximum, and minimum for questions 5, 6, 8, and 9 (See Appendix F). Table 2 displays the frequency and percentages of correct responses based upon grade level (See Appendix F). Table 3 displays the percentages of correct responses based upon gender (See Appendix F). Table 4 displays the frequency and percentages of various responses reported for question 6 (See Appendix F). Table 5 displays the frequency and percentages of various responses reported for question 8 (See Appendix F). Table 6 displays the frequency and percentages of various responses reported for question 9 (See AppendixF). Table 7presents the different responses that were recorded by participating students for question 7 (See Appendix F).

**Presentation of Findings**

Based on the overall results by grade level for questions 1 through 4, there was a clear disconnect between students’ understanding of the *x* and *y*-axes as equivalent to the independent and dependent variables of a function or graph which indicates a limited knowledge of correct terminology pertaining to graphs. This was shown by the significant decrease in correct responses for questions 1 and 2 as compared to questions 3 and 4. On average, 93.9% of students that participated in the survey correctly answered questions 1 and 2 while only 29.5% answered questions 3 correctly and 26.9% answered question 4 correctly. By looking at individual subject levels, it was evident that the gap between correctly identifying the *x* and *y*-axes and correctly identifying the independent and dependent variables decreased significantly as the grade level progressed. At the Algebra 1 level, 9.4% of students successfully identified the independent variable and 6.3% of students successfully identified the dependent variable while 84.3% on average correctly identified the *x* and *y*-axes. The success rate for these two criteria steadily increased with subject level to reach an average of 51.9% for both questions 3 and 4 by the Pre-Calculus level and an 84% at the Calculus level. These results demonstrated that the level of understanding of graphs and terminology is clarified significantly with each successive math class although it is not fully corrected by the time students reach the highest math offered at the high school level. Analyzing the data from a grade level or gender perspective shows that there was no evident correlation between students’ knowledge of identifying the required variables and their gender.

Responses to questions 5 and 6 revealed that students had increased difficulty in identifying a point of intersection when asked for the coordinate as opposed to identifying only one component of a point of intersection. On average, the success rate for question 5 fluctuated yet increased by content level while that of question 6 consistently increased by content level. The gap between the success rate for question 5 and 6 also narrowed as content level increased from 6.3% and 12.5% for Algebra 1 to 57.9% to 68.4% for Calculus. Individual component responses revealed that students tend to answer based on the unit interval divisions provided by the graph as shown by the most common response within the data for the point of interception of racers A and C, that of (60, 400). This is also supported by the most prominent response for question 6, that of 20 seconds. No significant correlation by grade level or gender was evident based on the data.

Survey question number 7 revealed that there appears to exist a better understanding of graphical interpretation with respects to rate among male students as opposed to female students as illustrated in the data. Overall, correct responses for this question steadily increased in conjunction with the content level with the exception of Algebra 2, which scored significantly higher than Calculus and Pre-Calculus. Responses for this question included a variety of correct and incorrect interpretations for the graph of runner C at a particular window of time(See Appendix F).These responses demonstrated that many students, especially at the lower levels, do not understand that there is a difference between a constant rate and a zero rate as shown by their interpretations of the zero slope for runner C at 13 seconds. Additionally, interpretation with respects to the context of the graph is also not clear for many students as shown by participants noting that the runner stopped yet not identifying a possible reason given that it was a hurdles race.

Responses to questions 8 and 9 highlighted a difficulty in correctly reading graphs while still demonstrating competence with interpretation. As shown in the data, 65.6% of Algebra 1 students were able to identify the winning runner based on the given graph and its context but only 28% of them were also able to determine the runner’s winning time. This again illustrates the issue of students estimating component values based on the labeled units. As with all other questions, the percent of correct student responses also showed a positive correlation with the subject level and the gap between correct responses for questions 8 and 9 also narrowed as content increased.

**Limitations of the Design**

Several limitations arose in the process of conducting this study due to time constraints and available survey participants. These limitations ranged from flaws within the survey to flaws with the study participant selections. Additionally, although a response key allowing for fluctuations in response was created, a discrepancy in the data results was apparent based on the scoring process.

The selection of classes to which the survey was given was not an ideal distribution of grade and performance levels. The only Algebra 1 and Geometry classes available for the study were comprised of strictly 8th grade students since the survey was carried out extremely close to the end of the high school year. As a result, our survey population only included one 9th grade student given that the next level of math, Algebra 2, rarely includes freshman students at the school from which we sourced this level of the study participants. Additionally, all but one of the classes selected for the study were mainstream level classes. Due to time constraints and previous exposure to the survey questions, the only available Pre-Calculus class that could be utilized in the survey was classified as an Honors class. This set of students had minimal to none previous exposure to the graph presented in the survey as opposed to the regular Pre-Calculus classes at the same site which had previous seen and responded to the graph and general question related to it.

The survey design also presented an issue that was only evident after the study had been carried out and the data collected. Questions 5, 6, and 9 required a numerical response which could not be answered with equal precision throughout the survey since the graph provided did not include grid lines. To adjust for fluctuation within readings of the graph, a range or window of correct responses was selected for each of these questions. This may have proved to be insufficient and could have possibly been fixed by simply providing the grid lines so as to serve as an easier scenario with respects to interpreting the numerical aspects of the survey. Several participant surveys displayed a common issue with simply drawing a straight line. Some surveys included the correct idea of horizontal and vertical lines to aid in identifying the required coordinate yet appeared not to be strictly horizontal or vertical and skewed towards the units labeled on the graph(See Appendix E).With the addition of grid lines, participants would most likely be more capable of correctly identifying a coordinate on the graph since it could reduce the

Fluctuation in numerical responses also accounted for another limitation as a result of the grading or scoring method that was selected for the data gathering and analysis process. Although the score key allowing for the fluctuation previously mention was created, it did not address questions that could be analyzed in two individual parts. This issue came to light when scoring question 5 in which participants were required to state a coordinate point with an *x* and *y* component. Some students failed to state a coordinate and simply stated only one component that may or may not have been correct if analyzed individually. Other participants correctly identified one component but not the other and so were scored as incorrect. Only those students whom successfully chose an *x* and *y* component within the window range for each part were recorded as correct responses. This issue was slightly addressed in the data analysis process by separating the components when calculating the basic statistics however it was not rectified when analyzing for correct responses.

**CONCLUSIONS AND RECOMMENDED FUTURE STUDIES**

As indicated from the above, this study looked at students’ ability to interpret and derive meaning from line graphs, by performing a simple survey about a graph in a real-world scenario. The purposes of this research is to see if our results agree or disagree with the current body of knowledge, which suggests that though students are often able to define and even draw graphs, they frequently perform poorly on graph interpretation items. The sample for this study consisted of 60 eighth-grade students primarily in Algebra I and Geometry classes, one ninth-grade student in Algebra II class, 28 eleventh-grade students in either Algebra II, Pre-calculus, or Calculus classes. There are also 29 twelfth-grade students ranging from Algebra II to Calculus, with the majority of these students in the Calculus class. Descriptive analyses of the finding in this simple research examined the patterns of students’ errors and the nature of their construction.

According to the result for question 1 and question 2, the preponderance of the participants in all grades successfully identify the x-axis represents the time in seconds and the y-axis represent the distance in meters. On the other hand, they did poorly on questions 3 and 4. The finding suggested that in this survey students cannot connect the x-axis as the independent variable and the y-axis as the dependent variable. As indicated in the previous section, these findings indicated that the level of understanding of graphs and terminology is clarified significantly with each successive math class although it is not fully corrected by the time students reach the highest math offered at the high school level. The outcomes of the successes rate for question 5 and question 6 are also dissimilar. In question 5, students were asked to identify the coordinate (time, distance) of the intersection of runners A and B. Nevertheless, question 6 simply asks for the time in which students need to provide the approximate time that runners A and C have the same distance. Overall, participants have greater number in answering question 6 correct than question 5. Moreover, the eighth-grade students in Algebra I have much lower score in both questions 5 and 6 in comparison to the twelfth-grade students in Calculus class, which is an expected outcome of the result. The finding also indicates that male and female participants did equally well in both of these questions.

Based on the result of this simple survey, the following areas are recommended for the modification of the survey and further research. First of all, the populations of the participants need to be randomly selected. We need to select the participants in a way that they are equally represented according to their grade-levels. The ideal population should include the main stream students, special education students, GATE students, socially disadvantaged students, and EL students. Participants in each of the subject should also be selected from several different classes with different teachers.

As stated in the Limitations of the Design section, there are some issues that are presented on the data collection processes. The surveyors might need to give clearer instructions to the participants and they also need to monitor carefully through the whole process of getting the data. Such as making sure that students label their grade levels and their genders. The questionnaires are acceptable, but the graph itself may need to be modified. Some students provided incorrect responses to the questions, because they did not sketch straight lines when they looked for the coordinate in question 5 or the time in question 6. Therefore, the graph should have the vertical and horizontal grid lines so that students would be more likely to demonstrate their factual abilities in understanding and interpreting the graph.

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**APPENDICES**

Appendix A

**INFORMED CONSENT**

The study in which you are being asked to participate is designed to investigate the extent to which middle and high school students struggle with interpretations of graphs and the relationship this has on the most commonly held graphing misconceptions among these students. This study is being conducted by Alyshea Corsaro, Ky Le, Claudia Preciado, and Swinton Sama under the supervision of Prof. Enrique G. Murillo Jr., Professor of Educational Psychology and Counseling, California State University, San Bernardino. This study has been approved by the Institutional Review Board, California State University, San Bernardino.

**PURPOSE:** To study to what extent do middle and high school students struggle with interpreting graphs, and what are the most commonly held graphing misconceptions amongst these students.

**DESCRIPTION:** Participants chosen from the middle and high school level will be given a survey in the form of a series of questions relating to graphing and graphical representations and interpretations.

**PARTICIPATION:** Participation in this study is voluntary and may be discontinued at any time and will involve no penalty to a participant’s current grade.

**CONFIDENTIALITY OR ANONYMITY:** This study is anonymous and will not require participants to identify themselves on the survey they will be given.

**DURATION:** Subjects in this study are expected to participate over one class period lasting roughly 57 minutes.

**RISKS:** There are no foreseeable risks to participants involved in this study.

**BENEFITS:** There are no apparent benefits for the study participants.

**VIDEO/AUDIO/PHOTOGRAPH:** I understand that this research will be photographed Initials\_\_\_.

**CONTACT:** Prof. Enrique G. Murillo Jr., Professor of Educational Psychology and Counseling, California State University, San Bernardino may be contacted in case of questions pertaining to this research via email at [**emurillo@csusb.edu**](http://emurillo.org/email.htm) or by phone at (909) 537-5632 .

**RESULTS:** Results of this study may be obtained through group project submissions for EDUC 607 Spring 2012.

Appendix B

Math Class: ­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Grade Level: \_\_\_\_\_\_\_ Circle Gender: M / F

**“HURDLES RACE”**

400

60

Time (Seconds)

Distance (Meters)

A

B

C

**The graph sketched above describes what happened when 3 athletes A, B, and C enter a 400 meter hurdle race. Answer the following questions to the best of your ability based on the graph.**

1. What is the *x*-axis?
2. What is the *y*-axis?
3. What is the independent variable?
4. What is the dependent variable?
5. What is the point of intersection of runners A and B?
6. Approximately at what time do racers A and C have the same distance?
7. What do you think happened to runner C at approximately 13 seconds?
8. Who won the race?
9. What was the winning runner’s final time?

Appendix C

Math Class: ­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Grade Level: \_\_\_\_\_\_\_ Circle Gender: M / F

**“HURDLES RACE”**

400

60

Time (Seconds)

Distance (Meters)

A

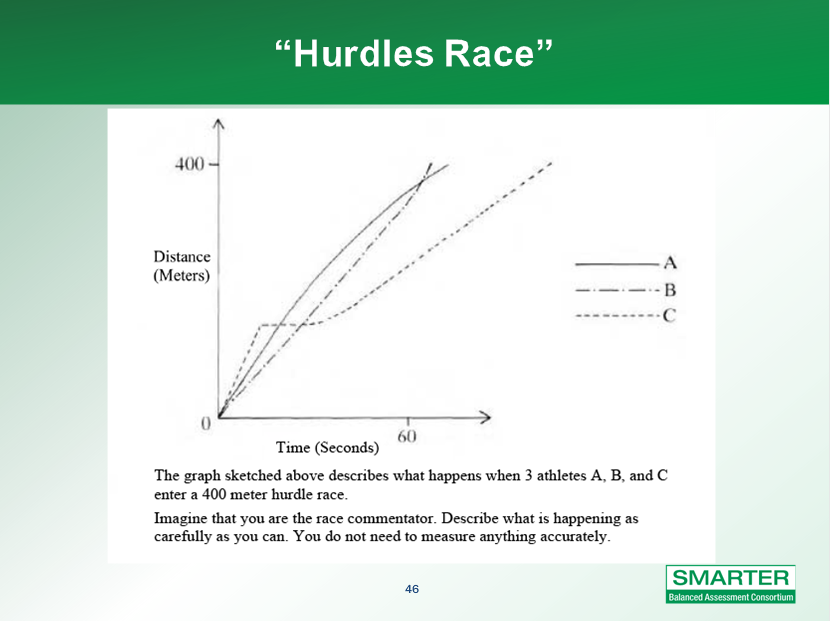
B

C

**The graph sketched above describes what happened when 3 athletes A, B, and C enter a 400 meter hurdle race. Answer the following questions to the best of your ability based on the graph.**

1. What is the *x*-axis? Time (Seconds)
2. What is the *y*-axis? Distance (Meters)
3. What is the independent variable? Time (Seconds)
4. What is the dependent variable? Distance (Meters)
5. What is the point of intersection of runners A and B? (65-69, 380-395)
6. Approximately at what time do racers A and C have the same distance?18-22 Seconds
7. What do you think happened to runner C at approximately 13 seconds? Responses will vary (create a list of common responses)
8. Who won the race? B
9. What was the winning runner’s final time? 66-70 Seconds

Appendix D



Appendix E

Appendix F

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1: Survey Statistics | | | | | | | | | | | | | | | | | |
|  | | | | | Q5x | | | Q5y | | | Q6 | | | Q8 | | | Q9 |
| N | | Valid | | | 141 | | | 141 | | | 141 | | | 141 | | | 141 |
| Missing | | | 30 | | | 30 | | | 30 | | | 30 | | | 30 |
| Mean | | | | | 61.67 | | | 389.21 | | | 24.62 | | | 2.11 | | | 70.37 |
| Median | | | | | 60.00 | | | 390.00 | | | 20.00 | | | 2.00 | | | 69.00 |
| Mode | | | | | 60 | | | 400 | | | 20 | | | 2 | | | 70 |
| Range | | | | | 19 | | | 50 | | | 60 | | | 2 | | | 92 |
| Minimum | | | | | 50 | | | 350 | | | 10 | | | 1 | | | 38 |
| Maximum | | | | | 69 | | | 400 | | | 70 | | | 3 | | | 130 |
| Table 2: Statistics by Grade Level | | | | | | | | | | | | | | | |
|  | | | | Frequency | | | Percent | | | Valid Percent | | | Cumulative Percent | | |
| Valid | Grade 8 | | | 60 | | | 35.1 | | | 42.6 | | | 42.6 | | |
| Grade 9 | | | 1 | | | .6 | | | .7 | | | 43.3 | | |
| Grade 10 | | | 23 | | | 13.5 | | | 16.3 | | | 59.6 | | |
| Grade 11 | | | 28 | | | 16.4 | | | 19.9 | | | 79.4 | | |
| Grade 12 | | | 29 | | | 17.0 | | | 20.6 | | | 100.0 | | |
| Total | | | 141 | | | 82.5 | | | 100.0 | | |  | | |
| Missing | System | | | 30 | | | 17.5 | | |  | | |  | | |
| Total | | | | 171 | | | 100.0 | | |  | | |  | | |
| Table 3: Statistics by Gender | | | | | | | | | | | | | | |
|  | | | Frequency | | | Percent | | | Valid Percent | | | Cumulative Percent | | |
| Valid | Male | | 65 | | | 38.0 | | | 46.1 | | | 46.1 | | |
| Female | | 76 | | | 44.4 | | | 53.9 | | | 100.0 | | |
| Total | | 141 | | | 82.5 | | | 100.0 | | |  | | |
| Missing | System | | 30 | | | 17.5 | | |  | | |  | | |
| Total | | | 171 | | | 100.0 | | |  | | |  | | |

Appendix F

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 4: Question 6 | | | | | |
|  | | Frequency | Percent | Valid Percent | Cumulative Percent |
| Valid | 10 | 6 | 3.5 | 4.3 | 4.3 |
| 15 | 12 | 7.0 | 8.5 | 12.8 |
| 16 | 4 | 2.3 | 2.8 | 15.6 |
| 17 | 2 | 1.2 | 1.4 | 17.0 |
| 18 | 7 | 4.1 | 5.0 | 22.0 |
| 19 | 5 | 2.9 | 3.5 | 25.5 |
| 20 | 71 | 41.5 | 50.4 | 75.9 |
| 21 | 1 | .6 | .7 | 76.6 |
| 22 | 1 | .6 | .7 | 77.3 |
| 25 | 4 | 2.3 | 2.8 | 80.1 |
| 30 | 9 | 5.3 | 6.4 | 86.5 |
| 40 | 3 | 1.8 | 2.1 | 88.7 |
| 50 | 6 | 3.5 | 4.3 | 92.9 |
| 60 | 4 | 2.3 | 2.8 | 95.7 |
| 70 | 6 | 3.5 | 4.3 | 100.0 |
| Total | 141 | 82.5 | 100.0 |  |
| Missing | System | 30 | 17.5 |  |  |
| Total | | 171 | 100.0 |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 5: Question 8 Statistics | | | | | |
|  | | Frequency | Percent | Valid Percent | Cumulative Percent |
| Valid | A | 8 | 4.7 | 5.7 | 5.7 |
| B | 110 | 64.3 | 78.0 | 83.7 |
| C | 23 | 13.5 | 16.3 | 100.0 |
| Total | 141 | 82.5 | 100.0 |  |
| Missing | System | 30 | 17.5 |  |  |
| Total | | 171 | 100.0 |  |  |

Appendix F

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 6: Question 9 Statistics | | | | | |
|  | | Frequency | Percent | Valid Percent | Cumulative Percent |
| Valid | 38 | 1 | .6 | .7 | .7 |
| 55 | 1 | .6 | .7 | 1.4 |
| 60 | 16 | 9.4 | 11.3 | 12.8 |
| 62 | 1 | .6 | .7 | 13.5 |
| 64 | 2 | 1.2 | 1.4 | 14.9 |
| 65 | 33 | 19.3 | 23.4 | 38.3 |
| 66 | 3 | 1.8 | 2.1 | 40.4 |
| 67 | 1 | .6 | .7 | 41.1 |
| 68 | 11 | 6.4 | 7.8 | 48.9 |
| 69 | 2 | 1.2 | 1.4 | 50.4 |
| 70 | 51 | 29.8 | 36.2 | 86.5 |
| 73 | 2 | 1.2 | 1.4 | 87.9 |
| 75 | 1 | .6 | .7 | 88.7 |
| 80 | 1 | .6 | .7 | 89.4 |
| 90 | 4 | 2.3 | 2.8 | 92.2 |
| 95 | 1 | .6 | .7 | 92.9 |
| 97 | 1 | .6 | .7 | 93.6 |
| 100 | 5 | 2.9 | 3.5 | 97.2 |
| 110 | 3 | 1.8 | 2.1 | 99.3 |
| 130 | 1 | .6 | .7 | 100.0 |
| Total | 141 | 82.5 | 100.0 |  |
| Missing | System | 30 | 17.5 |  |  |
| Total | | 171 | 100.0 |  |  |

Appendix F

Table 7: Response Variations for Question 7

|  |  |
| --- | --- |
| Participant Response | Number of Responses |
| Speed up | 3 |
| Sped up got tired and slowed down | 32 |
| Stopped running/Took a break | 48 |
| Tripped/Fell | 25 |
| Ran at a constant speed | 11 |
| Got dehydrated | 3 |
| Got up and kept running | 1 |
| Stayed at the same distance/place | 5 |
| Went the wrong way | 1 |
| I don’t know | 11 |